MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

EMT1026 – ENGINEERING MATHEMATICS 2

(All Sections / Groups)

2nd MARCH 2020
9:00 AM – 11:00 AM
(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **6 pages** (including cover page) with **4** Questions only.
2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.
5. Several tables are provided in the Appendix for your reference.

Question 1

(a) Let $f(x,y)=\dfrac{(x+y)^2}{x^2+y^2},(x,y)\neq(0,0)$.

(i) Find $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along the y-axis. [2 marks]

(ii) Find $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along the line $y=x$. [2 marks]

(iii) What can you conclude about $\lim\limits_{(x,y)\to(0,0)} f(x,y)$? [2 marks]

(b) Let $g(x,y)=1-x-y+xy$. Suppose D is the region bounded by the parabola $y=x^2$ and the line $y=4$, find the absolute maximum and minimum values of $g(x,y)$ on the set D.

[9 marks]

(c) Using the method of Lagrange multipliers, find the maximum and minimum values of $f(x,y,z)=x+y+2z$ on the sphere $x^2+y^2+z^2=3$.

[10 marks]

Question 2

(a) Find the complete solution for the following differential equation:

$$y''-5y'+4y=3e^{2x}-7\cos 2x$$

[12 marks]

(b) Using the power series method, find the first five nonzero terms of the solution for the following differential equation:

$$(x-5)y'+y=0$$

[13 marks]

Continued...

Question 3

(**Note**: The tables in APPENDIX A and APPENDIX B may be useful for solving this question.)

(a) Three time-domain functions are related by $y(t)=x(t)*h(t)$, where $*$ denotes convolution. Let $X(s)$, $Y(s)$ and $H(s)$ be the Laplace transforms of $x(t)$, $y(t)$ and $h(t)$, respectively.

(i) If $x(t)=\cos at-2\sin at$ and $h(t)=e^{at}\sinh at$, find $X(s)$ and $H(s)$. Then show that

$$Y(s)=\frac{a}{s\left(s^2+a^2\right)}.$$

[7 marks]

(ii) Hence, or otherwise, determine an expression for $y(t)$.

[3 marks]

(iii) If $y(t)=t\sinh 3t$, find $Y(s)$. Furthermore, if $H(s)=\frac{1}{s+3}$, show that

$$X(s)=\frac{6s}{(s-3)^2(s+3)}.$$

[6 marks]

(b) Consider the following ordinary differential equation (ODE):

$x''(t)-6x'(t)+9x(t)=-18e^{-3t}$, subject to $x(0)=0$ and $x'(0)=6$.

(i) By taking Laplace transforms, show that $L\{x(t)\}$ has the same expression as $X(s)$ in (a)(iii).

[5 marks]

(ii) Hence, or otherwise, determine the complete solution to the ODE.

[4 marks]

Continued...

Question 4

A thin metal rod lying on the x-axis has its ends located at $x = 0$ and $x = L$. The left end is kept at temperature α and the right end at temperature β. Initially, the temperature across the rod is given by $f(x)$.

(a) Set up the initial-boundary value problem (IBVP) that governs $u(x,t)$, the temperature across the rod at any time $t > 0$.

[4 marks]

(b) If $\alpha = 0$ and $\beta = 0$, use the method of separation of variables to show that the IBVP in (a) has the **general solution**

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) e^{-\left(\frac{n\pi}{L}\right)^2 kt},$$

where b_n are constants to be determined and k is the thermal diffusivity of the rod.

[18 marks]

(c) If $\alpha = 1$ and $\beta = 3$, set up the ordinary differential equation that governs the temperature across the rod at **equilibrium/steady-state**.

[3 marks]

Continued...

APPENDIX A:

Common Laplace Transform Pairs

	$f(t)$	$F(s)=L\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^n	$\frac{n!}{s^{n+1}}$, $n=1,2,\ldots$
4.	e^{at}	$\frac{1}{s-a}$
5.	$\cos at$	$\frac{s}{s^2+a^2}$
6.	$\sin at$	$\frac{a}{s^2+a^2}$
7.	$\cosh at$	$\frac{s}{s^2-a^2}$
8.	$\sinh at$	$\frac{a}{s^2-a^2}$
9.	$u(t-a)$	$\frac{e^{-as}}{s}$
10.	$\delta(t-a)$	e^{-as}

Continued...

APPENDIX B:

Laplace Transform Properties

	Property Name	Formula
1.	Linearity	$L\{af(t)+bg(t)\}=aL\{f(t)\}+bL\{g(t)\}$
2.	s - shifting	$L\{e^{at}f(t)\}=F(s-a)$
3.	Transform of Derivative	$L\{f'(t)\}=sL\{f(t)\}-f(0)$ $L\{f''(t)\}=s^2L\{f(t)\}-sf(0)-f'(0)$
4.	Transform of Integration	$L\left\{\int_0^t f(\tau)d\tau\right\}=\frac{1}{s}F(s)$
5.	t - shifting	$L\{u(t-a)f(t-a)\}=e^{-as}F(s)$
6.	Differentiation of Transform	$L\{t.f(t)\}=-F'(s)$
7.	Integration of Transform	$L\left\{\frac{f(t)}{t}\right\}=\int_s^\infty F(\tilde{s})d\tilde{s}$
8.	Convolution Theorem	$L\{f(t)*g(t)\}=F(s).G(s)$, where $f(t)*g(t)=\int_0^t f(\tau)g(t-\tau)d\tau$.

End of paper.